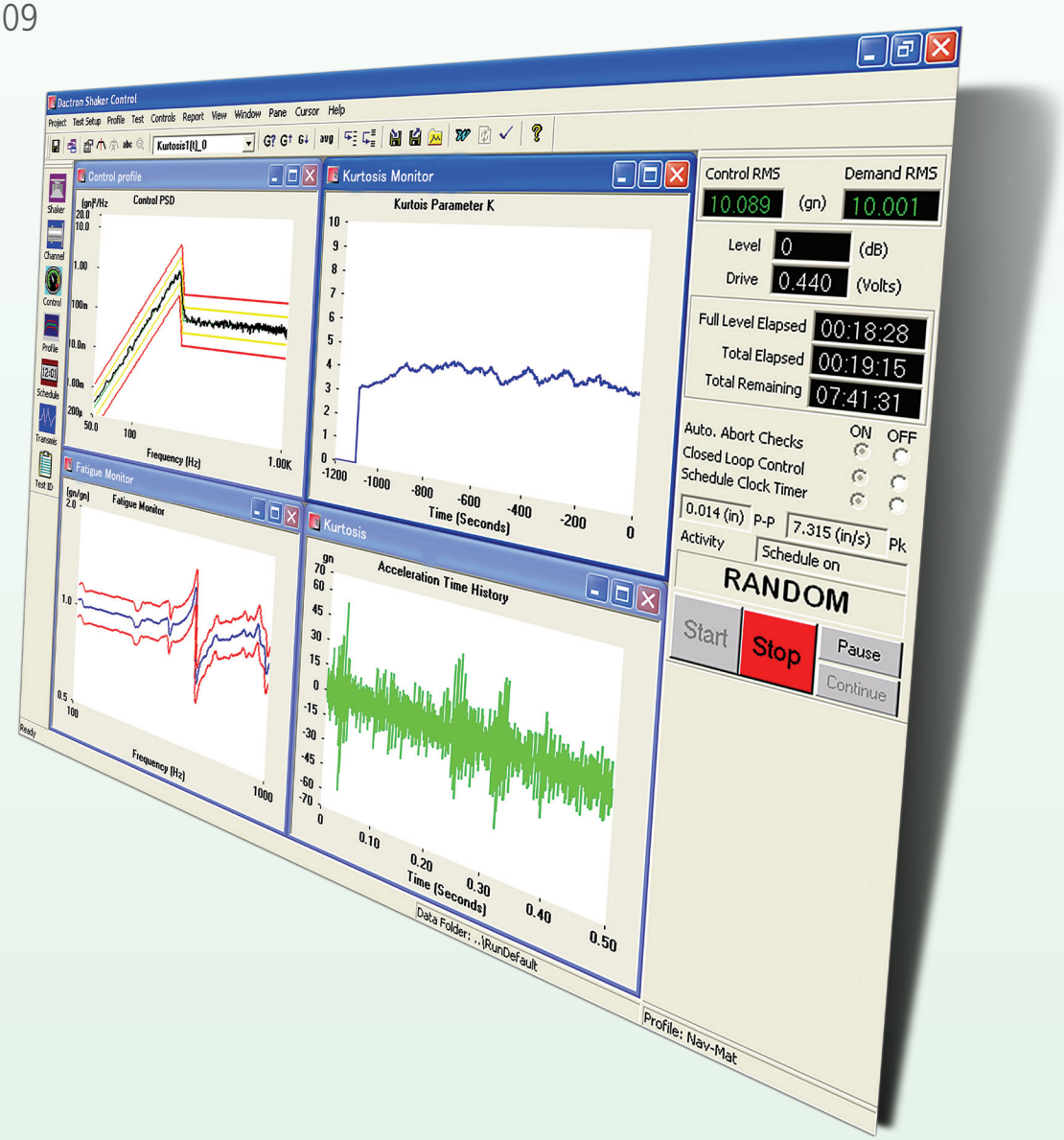
BRÜEL & KJÆR

KURTOSIS IN RANDOM VIBRATION CONTROL

September 2009



LDS A member of the Brüel & Kjær group

www.bksv.com/controllers

Brüel & Kjær
Incorporating LDS and Lochard

TABLE OF CONTENTS

KURTOSIS IN RANDOM VIBRATION CONTROL

What is Kurtosis? 3
Differences between Gaussian and K > 3 Signals 4
User Interface and Setup for Kurtosis 6

What is Kurtosis?

Kurtosis is a statistical parameter used to characterize a signal. In essence it provides a measure of the "peakedness" of a random signal. Signals that have a higher kurtosis value have more peaks that are greater than three-sigma; that is, peaks that are greater than three times the RMS value of the signal.

In the real world many kinds of vibration environments are characterized by signals that have high kurtosis value (relative to Gaussian random). The fatigue and damage potential for these vibrations are higher than for a pure Gaussian replication of the vibration environment. Hence using a traditional Gaussian random signal as the test signal will actually under test the product for its service environment.

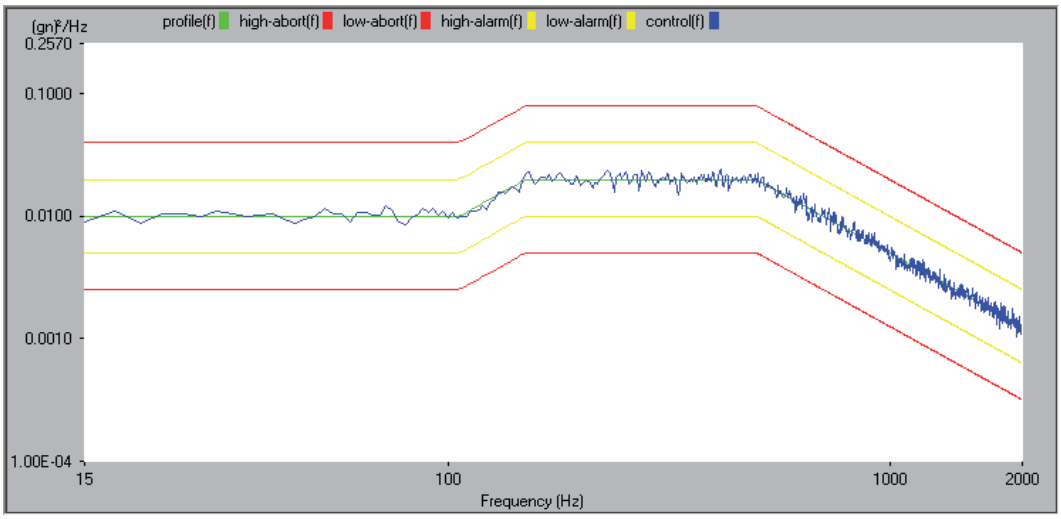
Kurtosis can be expressed as a normalized value "K" by dividing the fourth statistical moment divided by the square of the second statistical moment. The equation below shows the K calculation for N samples.

$$K = \frac{\frac{1}{N}\sum (x_i^4)}{\left(\frac{1}{N}\sum (x_i^2)\right)^2}$$

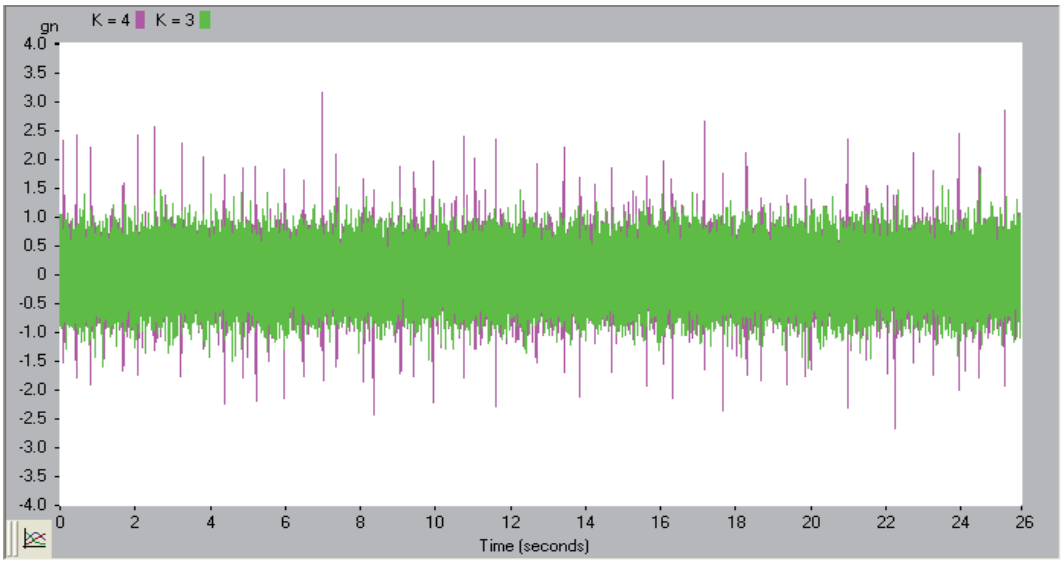
For a Gaussian signal the K value is always 3 regardless of the PSD shape or RMS level of the test profile.

Differences between Gaussian and K > 3 Signals

As a simple example of the differences between a Gaussian signal and a signal with a kurtosis value of greater than 3, consider the random test profile below. This profile has an RMS value of 4 g. Keep in mind that the shape of the profile PSD is not important in comparing signals with different kurtosis characteristics.

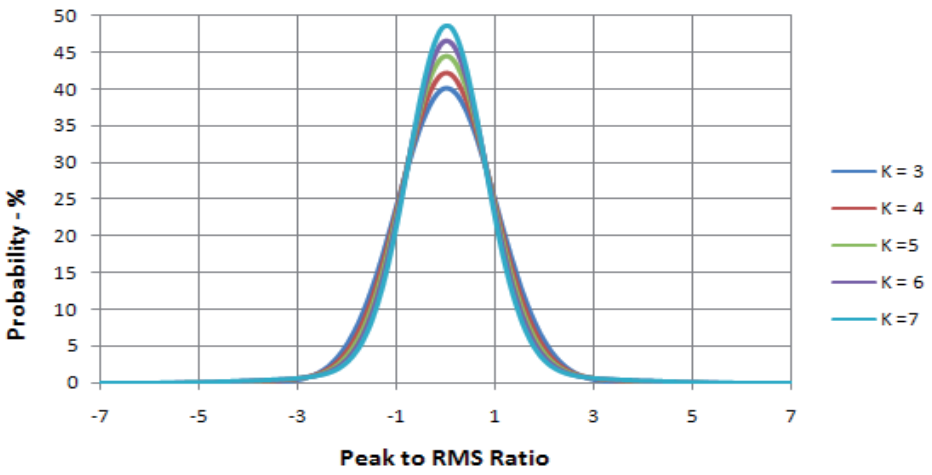


Since no difference will be seen in the frequency domain spectral data, we need to look in the time domain to see what effect kurtosis has on the test signal. The two plots below show the time histories for a test with K set to 3 (a pure Gaussian signal) and with K set to 4. With K = 4 the control signal spends more time at higher amplitude levels than does than a Gaussian signal.

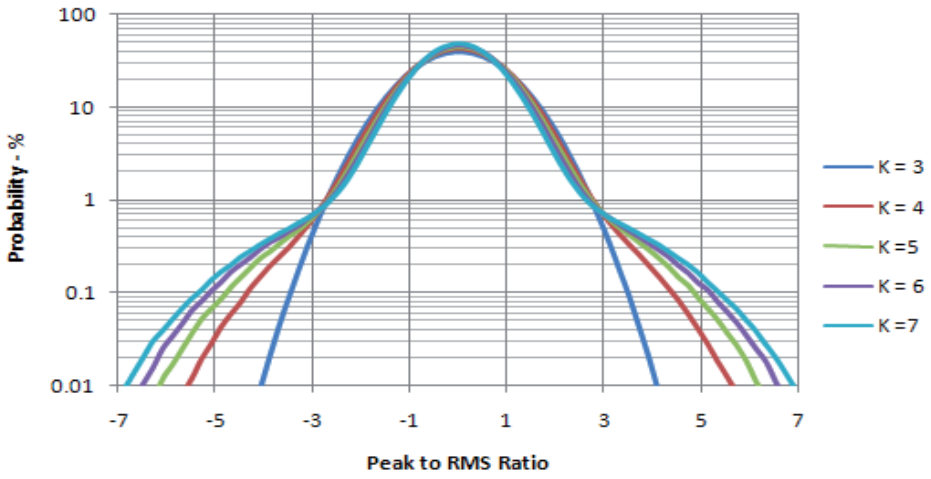


Comparison of the control signals for a Gaussian signal (Kurtosis K = 3) and a non-Gaussian signal with Kurtosis K = 4.

The fact that signals with higher kurtosis spend more time at higher amplitudes can be seen more clearly by comparing the histograms, or more properly the probability distribution functions (PDFs), of a Gaussian signal and signals with various kurtosis values. In plot below, the PDF of a Gaussian signal (K = 3) is compared to PDFs for signals with K values of 4, 5, 6 and 7. In the plot the Y scale represents the probability of occurrence of a given value and the X scale represents the peak to RMS ratio.



Since much of the difference in the distributions for different kurtosis values is in the "tails", it easier to compare the PDF plots with the Y plotted using a logarithmic scale.



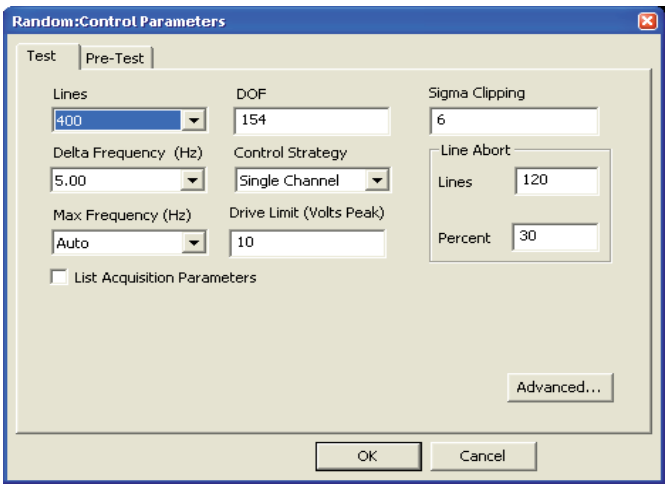
Measured Probability Distribution Functions for Kurtosis Values of 3 (Gaussian), 4, 5, 6 and 7.

User Interface and Setup for Kurtosis

Setting the kurtosis value for a test profile is easily done by the user in two steps.

Step 1 – Access the Control Parameter dialog.

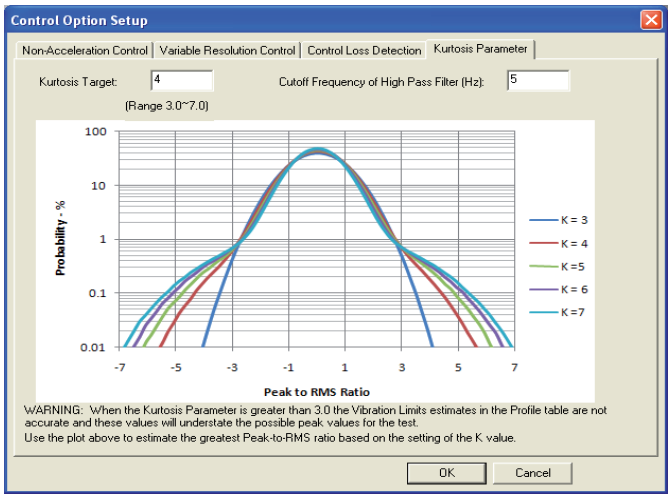
In this dialog the only action required is to click on the "Advanced" button in the lower right-hand side of the dialog panel.



Control Parameter dialog is used to access the "Advanced" settings.

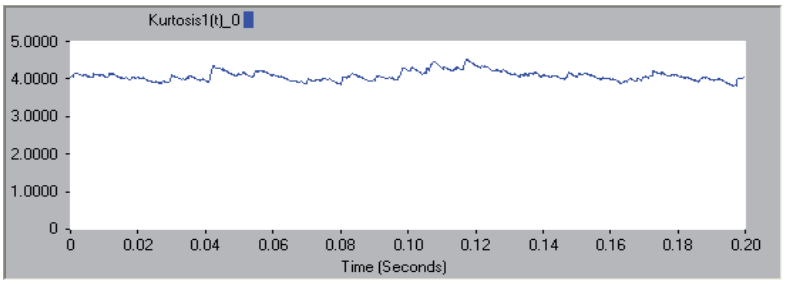
Step 2 – Set the Kurtosis Value.

The simply entry of a value from 3 to 7 in the "Kurtosis Target" field is the only user action required.



Setting "Kurtosis Target" or "K" value in the "Advanced" Control Parameter dialog.

A special "Kurtosis(t)" signal display provides an on-line verification that the test is achieving the desired K value. Since kurtosis is based on the fourth statistical moment there will be some variation in the K values as frames of data are acquired and the K value is calculated.



The control signal's kurtosis parameter can be monitored on-line using the special "Kurtosis(t)" signal display.

BO 0510 – 11

HEADQUARTERS: Brüel & Kjær Sound & Vibration Measurement A/S · DK-2850 Nærum · Denmark
Telephone: +45 77 41 20 00 · Fax: +45 45 80 14 05 · www.bksv.com · info@bksv.com
Local representatives and service organisations worldwide

Brüel & Kjær
Incorporating LDS and Lochard